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EFFICIENCY MEASUREMENT OF LEBANESE BANKS USING AN ELASTICITY-BASED PRODUCTION EFFICIENCY OPTIMIZATION MODEL (EPEOM)

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ABSTRACT. This study evaluates the efficiency of Lebanese commercial banks prior to the 2019 financial crisis, focusing on the structural weaknesses that contributed to the sector's collapse. Using data for the year 2018 for 19 banks and benchmarking against 7 international peers, the research introduces a new methodological framework that extends beyond traditional efficiency measurement techniques. It also addresses a gap in the literature by providing a pre-crisis quantitative assessment of Lebanese banks' performance. The study develops an Elasticity-Based Production Efficiency Optimization Model (EPEOM), which integrates production theory and substitution elasticities to assess bank performance. The results reveal significant inefficiencies driven by excessive exposure to government financing and poor resource allocation. These findings provide important insights for policymakers seeking to design effective banking sector restructuring strategies.

KEYWORDS: Lebanese Banks, Efficiency Measurement, Financial Crisis, Government Debt, Resource Allocation, Optimization Model, Production Theory, Elasticity of Substitution.

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Introduction

The Lebanese banking sector managed to continue operating during several periods of political instability by adapting to national, regional, and international laws, regulations, and financial agreements. In 2016, the Central Bank of Lebanon introduced financial engineering measures aimed at attracting fresh USD deposits from expatriates and foreign investors by offering high yields.[1] Most of these funds were placed with the central bank and used to maintain the peg of the Lebanese pound to the US dollar and to finance the government's budget deficit. In May 2019, deposit growth became negative for the first time in more than a decade, while the government faced increasing difficulty issuing new Eurobonds following multiple downgrades by international rating agencies. In October 2019, widespread protests and road closures disrupted economic activity and forced banks to suspend operations, triggering a liquidity crisis as depositors rushed to withdraw their funds. The situation deteriorated further in 2020 when Lebanon defaulted for the first time in its history on a $1.2 billion Eurobond payment[2].

Given Lebanon's financial crisis, a systematic assessment of bank performance in the immediate pre-crisis period is essential to fill a notable gap in the literature. Although the sector's collapse has been widely documented, there is a scarcity of empirical studies that

[1] URL: https://www.bdl.gov.lb/CB%20Com/Publications/Publications/BDLFinancialEngineering_Nov2016_EN.pdf
[2] URL: https://www.reuters.com/markets/rates-bonds/lebanons-financial-crisis-how-it-happened-2022-01-23/

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quantify how deficiencies in risk assessment and resource management contributed to this failure. To address this gap and improve the robustness of the analysis, this study aims to evaluate the efficiency of Lebanese banks in 2018 while conducting a benchmarking exercise against a selected group of international banks, thereby situating the performance of Lebanese institutions within a broader global context. This research departs from traditional approaches to bank efficiency analysis, such as ratio analysis, regression techniques, and frontier-based methods, by introducing a novel methodological framework that offers deeper and more comprehensive insights into bank performance.

The originality of this research lies in the development of a new mathematical optimization model designed to evaluate and improve bank performance. Grounded in production theory and efficiency analysis, the model treats banks as entities that transform inputs into financial outputs while incorporating elasticities of substitution and weighted production plans. By estimating these relationships, the model constructs an integrated performance indicator that provides a comprehensive measure of efficiency and captures the underlying economic interactions between resources and outputs.

Literature Review

Banks operate as financial intermediaries that transfer funds between depositors and borrowers while generating returns. Investors and depositors depend on the stability and performance of banks, making the evaluation of bank efficiency essential to ensure proper risk management and effective resource allocation. Traditionally, ratio analysis has been a widely employed tool for assessing banks' performance, focusing on indicators such as liquidity, profitability, leverage, and risk ratios. Financial ratios offer simplicity and facilitate benchmarking across institutions; however, numerous studies have highlighted their limitations in complex banking environments. Specifically, ratio analysis does not fully account for factors such as economies of scale or provide a comprehensive measure of overall performance (Yang, 2009). Ratios are primarily effective during the initial stages of performance evaluation (Whittington, 1980), and interpreting them in banking contexts can be challenging (Sherman & Gold, 1985). Despite these limitations, financial ratios underpin the CAMELS rating system, which evaluates banks based on capital adequacy, asset quality, management efficiency, earnings quality, liquidity, and sensitivity to market risk. This framework has been applied to analyze the financial stability of banks in Eastern Europe (Sargu, Roman, 2013). Regression analysis is another widely used econometric method that examines the relationship between one output variable and one or more input variables to estimate efficiency. However, despite allowing multiple inputs, it is limited by typically considering only a single output for a given set of inputs. To overcome these limitations, frontier efficiency approaches were introduced, classifying efficiency measurement methods into parametric and non-parametric models (Farrell, 1957). The parametric approach includes Stochastic Frontier Analysis, developed by Aigner (Aigner, Lovell, Schmidt, 1977), while the non-parametric approach encompasses Data Envelopment Analysis, introduced by Charnes, Cooper, and Rhodes (Charnes, Cooper, Rhodes, 1978). Many empirical studies have used frontier approaches to evaluate banking efficiency across different countries. Oral et al. assessed the efficiency of Turkish bank branches using Data Envelopment Analysis (DEA) (Oral, Yolalan, 1990), Al-Faraj et al. studied bank branch efficiency in Saudi Arabia using DEA (Al-Faraj, Alidi, 1993), and Vassэloglou et al. evaluated the relative efficiency of commercial banks in Greece using DEA (Vassэloglou, Gэokas, 1990). In the Middle East region, Ajlouni et al. applied DEA to measure bank efficiency (Ajlouni, Hmedat, Hemdat, 2011), while Єahin et al. employed DEA to examine the impact of the global crisis on the Turkish banking sector (Єahin, Gцkdemir, Цztьrk, 2016), focusing on factors such as bank size, ownership structure, and capitalization.

In contrast, empirical studies on Lebanese banks are limited. Saad and Moussawi measured bank efficiency using both DEA and Stochastic Frontier Analysis (SFA) (Saad, Moussawi, 2009), while Zreika et al. used DEA to assess efficiency changes following the global financial crisis (Zreika, Elkanj, 2011). Osman et al. also applied DEA to evaluate the relative performance of Lebanese banks (Osman, Hitti, Al-Ayoubi, 2008). To address the existing gap in the literature, this research proposes a new methodological framework for evaluating bank efficiency. Instead of relying solely on traditional tools such as ratio analysis, regression models, or conventional frontier techniques, the study develops a new mathematical optimization model grounded in production theory. The model treats banks as entities that transform inputs into outputs and incorporates concepts such as elasticities of substitution and weighted averages of production plans to construct an integrated performance indicator for evaluating bank efficiency.

Methodology and Data Collection

This study adopts a quantitative empirical methodology to evaluate the efficiency of Lebanese banks and to introduce a new optimization-based performance model. The empirical analysis relies on secondary data collected from Lebanese banks' annual reports, the Banque du Liban, the Ministry of Finance Lebanon, and international financial databases. The efficiency measurement focuses on the year 2018 and covers a representative sample of 19 Lebanese banks, whose combined assets represent 95% of the sector's total assets, together with 7 selected international peer banks used for benchmarking purposes. The quantitative framework of the study is based on a newly developed logarithmic optimization model grounded in production theory and efficiency analysis. In this framework, banks are modeled as economic entities that transform multiple inputs into financial outputs, while the feasible production set is assumed to be convex. The model introduces a logarithmic transformation of the objective function to capture multiplicative efficiency relationships and proportional variations between inputs and outputs more effectively. In addition, the model incorporates estimated elasticities of substitution, allowing flexible substitution among resources and outputs while reflecting the economic structure of banking production. A system of linear programming equations is integrated into the model to estimate parameters and compute a performance indicator, while slack variables capture deviations from optimal production. The model uses foreign banks as benchmarks to expand the production set, enabling a more comprehensive evaluation of Lebanese banks' efficiency.

Selecting and classifying variables as inputs or outputs is a critical step in efficiency analysis because there are no universally ideal criteria or model for determining the appropriate number or type of variables, researchers must define selection criteria consistent with the objectives of the study and the characteristics of the decision-making units (DMUs). In addition, the number of variables must remain compatible with the sample size to preserve the discriminating power of the model. According to Cooper et al., the number of DMUs (n) selected should be greater than or equal to the maximum between; three times the sum of inputs (m) and outputs (r), and their product $m \times r$ that is $n \geq \max\left(m \times r, 3\left(m+r\right)\right)$ (Cooper, Seiford, Tone, 2006). This constraint helps prevent the use of excessive variables, which could artificially inflate the number of efficient units and reduce the robustness of the results.

Various methods have been developed for selecting input and output variables. Ruiz et al. chose relevant variables based on their contribution to efficiency (Ruiz, Pastor, Sirvent, 2002). Jenkins et al. introduced a multivariate statistical approach to reduce the number of variables using partial correlation (Jenkins, Anderson, 2003). Ruggiero used regression analysis techniques to identify the relevant variables (Ruggiero, 2005). Morita and Haba selected the input-output variables using a 2-level orthogonal layout experiment (Morita, Haba, 2005). Edirisinghe and Zhang proposed a generalized DEA approach to select inputs and outputs (Edirisinghe, Zhang, 2007). Morita and Avkiran selected the best combination of inputs and outputs using a 3-level orthogonal layout design (Morita, Avkiran, 2009).

To determine the most appropriate variables, this study selected eight commonly used banking indicators based on the literature. Data were collected from 19 Lebanese banks over the period 2014–2018 and from seven international peer banks for 2018, with all amounts converted to USD using prevailing exchange rates. The selected variables include Total Assets

(TA), Customer Deposits (CD), Cash & Deposits at the Central Bank (CD_CB), Investments (INV), Loans & Advances to customers (LAC), Net Interest Income (NII), Net Profit (NP), and Net Fees & Commissions (NFC). A correlation analysis using SPSS[1] was conducted on Lebanese banks over the five-year period (2014–2018) to examine relationships among these variables and identify the appropriate classification of inputs and outputs. This analysis helped eliminate redundant indicators, ensure meaningful input–output relationships, and account for the significant exposure of Lebanese banks to government debt in the variable selection process.

Total Assets is strongly positively correlated with Cash and Deposits at the Central Bank (0.9333) and with Customer Deposits (0.7461), while Customer Deposits are strongly correlated with Loans and Advances to Customers (0.7038). Investments show weak or negative correlations with most variables, notably a negative correlation with CD_CB of -0.2883, indicating limited linear relationships. Net Interest Income is moderately correlated with TA (0.5758) and CD_CB (0.5584), and Net Profit correlates with NII (0.5333) and LAC (0.3806), reflecting their influence on profitability. Net Fees and Commissions have a low linear impact on other variables. Correlations between any pair of inputs or outputs should not be too high, as this may indicate proxy relationships, and each output must correlate with at least one input to ensure the model is properly specified and yields meaningful results.

Accordingly, this study is implemented using the Elasticity-Based Production Efficiency Optimization Model (EPEOM), specifically the industry-level optimization model in its logarithmic form, with three inputs; Customer Deposits, Cash and Deposits at the Central Bank, and Investments, and two outputs; Loans and Advances to Customers, and Net Profit. The average correlation among the selected variables, together with the defined inputs and outputs and their corresponding data in USD for the year 2018 across the 26 Lebanese and international banks, are presented in the following tables:

Table 1

Labels of Inputs and Outputs

Output 1	Output 2	Input 1	Input 2	Input 3
Loans & Advances to Customers	Net Profits	Customer Deposits	Cash & Deposits at Central Bank	Investments

Source: compiled by the author

Table 2

Average correlations between the eight variables

Correlations	TA	CD	CD_CB	INV	LA&C	NII	NP	NFC
TA	1							
CD	0.7461	1						
CD_CB	0.9333	0.686	1					
INV	-0.0737	-0.0967	-0.2883	1				
LA&C	0.4706	0.7038	0.4538	-0.1982	1			
NII	0.5758	0.4551	0.5584	-0.0721	0.2820	1		
NP	0.3992	0.3538	0.3592	0.1799	0.3806	0.5333	1	
NFC	0.1803	0.2643	0.1663	-0.1545	0.332	0.099	0.0246	1

Source: compiled by the author using SPSS Software

Table 3

Inputs/outputs values for the Lebanese and Foreign banks in 2018

In Million USD					
Bank	Input_1	Input_2	Input_3	Output_1	Output_2
Audi	31956	16447	12922	13267	501
BBAC	6315	2898	2626	1751	51.7
BEMO	1307	483	629	728	18.9
BLC	3908	1114	1820	1607	41
BLF	11315	4076	2934	4323	80.6
BLOM	26917	21980	5165	7165	512
BML	1416	817	722	401	20
BOB	13346	4735	5966	5759	207
Byblos	18467	10829	5370	5442	164
Cedrus	877	693	988	204	43.2
Credit Bank	3338	1776	206	1958	22.58
Credit Libanais	9340	4915	3177	3304	83.36
Fenicia	1432	572	533	523	14.8
FNB	3872	1713	1595	947	36.75
FB	17023	6088	6200	6740	174
IBL	5860	2503	3009	873	113
MEAB	1866	410	292	1367	4.9
MED	13334	5479	7670	3619	31
SGBL	18699	11022	6389	5810	195
City Group (USA)	1013000	188000	359000	672000	18000
Societe Generale (France)	476798	110484	502734	511585	5212
Deutsche Bank (Germany)	645623	215889	715401	457900	390
Llyods (UK)	532449	69619	263558	617515	5604
Al Rajhi (Saudi Arabia)	78376	11532	11483	62417	2746
Halk Bank (Turkey)	41645	5878	11608	49058	512
NBE (Egypt)	60633	24227	14795	28364	565

Note: Amounts are converted to USD using the exchange rate prevailing on the date corresponding to each balance sheet figure.

Source: compiled by the author based on the Banks' Annual Reports

Model Formulation and Empirical Results

This research introduces a new mathematical optimization model, titled «Elasticity-Based Production Efficiency Optimization Model (EPEOM)», to enhance the valuation of bank performance. Building upon production theory and efficiency analysis, the model incorporates the estimation of elasticities of substitution between inputs and outputs to provide a more comprehensive and realistic assessment of operational efficiency. By using weighted averages of production plans and optimizing resource allocation, the model allows for deeper insights into bank performance beyond traditional DEA approaches. The following subsections present the detailed mathematical formulation of the model and the results obtained from its application to the dataset.

Let's denote:

m: the number of resources or inputs (m = 3);

$x = (x_1,\dots , x_m)$ is the vector of resources (input indicators);

s: the number of products or outputs ($s = 2$);

$y = (y_1,\dots , y_s)$ is the vector of products (output indicators);

n: The number of Lebanese and foreign banks covered in this research (n = 26);

n_d: The number of domestic (Lebanese) banks ($n_d \le n$, n_d=19).

For each bank, the input and output indicators are denoted:

$\bar{x}_{ri}$ is the i^{th} "resource: of the r^{th} bank for $r=\overline{1,n}$, $i=\overline{1,m}$;

$\bar{y}_{rj}$ is the j^{th} "product" of the r^{th} bank for $r=\overline{1,n}$, $j=\overline{1,s}$.

Let's denote:

$\bar{x}_r := (\bar{x}_{1r},\ldots,\bar{x}_{mr})$ Represents the vector of resources of the r^{th} bank for $r=\overline{1,n}$;

$\bar{y}_r := (\bar{y}_{1r},\ldots,\bar{y}_{sr})$ Represents vector of products of r^{th} bank for $r=\overline{1,n}$;

$(\bar{x}_r,\bar{y}_r)$: The "production" plan of the r^{th} bank for $r=\overline{1,n}$.

The model assumes that any weighted average plan $(\tilde{x},\tilde{y})$ of individual plans $(\bar{x}_r,\bar{y}_r)$ is itself a feasible production plan; this relationship is expressed as:

$$(\tilde{x},\tilde{y})=\sum_{r=1}^{n}\lambda_r\cdot(\bar{x}_r,\bar{y}_r), \tag{1}$$

where $\lambda_r \geq 0$, $r=\overline{1,n}$, and $\sum_{r=1}^{n}\lambda_r=1$.

Let us denote by $\tilde{\Omega}$ the set of all (feasible) weighted average plans $(\tilde{x},\tilde{y})$:

$$\tilde{\Omega}:=\left\{\sum_{r=1}^{n}\lambda_r\cdot(\bar{x}_r,\bar{y}_r)\;\middle|\;\lambda_r\geq 0,\ \sum_{r=1}^{n}\lambda_r=1\right\}. \tag{2}$$

Note that $\tilde{\Omega}$ is the convex hull of the set of plans $(\bar{x}_r,\bar{y}_r)$, $r=\overline{1,n}$. It is natural to assume that any plan obtained by increasing the size of resources and reducing the size of products, in a feasible production plan $(\tilde{x},\tilde{y})$, is also a feasible plan. Based on this reasoning, we can define the set Ω of all feasible production plans as follows:

$$\Omega:=\left\{(x,y)\;\middle|\;\exists(\tilde{x},\tilde{y})\in\tilde{\Omega}:\ x\geq\tilde{x}, y\leq\tilde{y}\right\}. \tag{3}$$

Under (2) and (3) assumptions, the plan (x,y) is feasible [i.e. $(x,y)\in\Omega$], If and Only If there exist values of λ_r, $r=\overline{1,n}$, such that

$$\sum_{r=1}^{n}\lambda_r\bar{x}_r \leq x, \tag{4}$$

$$\sum_{r=1}^{n}\lambda_r\bar{y}_r \geq y, \tag{5}$$

$$\sum_{r=1}^{n}\lambda_r=1, \tag{6}$$

$$\lambda_r\geq 0, \qquad r=\overline{1,n}. \tag{7}$$

It is reasonable to assume that experts can estimate the elasticities of substitution between different resources and products. Based on these estimates, parameters α_i and β_j can be calculated for the following integral performance indicator:

$$u(x,y) := \frac{\prod_{j=1}^{s} y_j^{\beta_j}}{\prod_{i=1}^{m} x_i^{\alpha_i}}, \tag{8}$$

where $x=(x_1,\ldots,x_m)$ and $y=(y_1,\ldots,y_s)$.

The function $u(x,y)$ has:

- **Constant Elasticities of Substitution:** $u(x,y)$ has constant elasticities of substitution between its variables x_i and y_j.
- **Logarithmic Maximization Equivalence:** maximizing $u(x,y)$ is equivalent to the maximization of its logarithm:

$$\ln u(x,y) = \sum_{j=1}^{s} \beta_j \ln y_j - \sum_{i=1}^{m} \alpha_i \ln x_i \, . \tag{9}$$

- **Alternative Linear Maximization:** alternatively, we can maximize a linear integral performance indicator of the form:

$$u(x,y) := \sum_{j=1}^{s} b_j y_j - \sum_{i=1}^{m} a_i x_i \, . \tag{10}$$

The function defined in (10) has constant rates of substitution, which can be estimated by experts; based on these estimates, the parameters a_i and b_j can be calculated. However, this model will not be implemented in this research.

Let us denote:

$$X := \sum_{r=1}^{n_d} x_r \, , \tag{11}$$

$$Y := \sum_{r=1}^{n_d} y_r \, , \tag{12}$$

$$U(X,Y) := \frac{\prod_{j=1}^{s} Y_j^{\beta_j}}{\prod_{i=1}^{m} X_i^{\alpha_i}} . \tag{13}$$

The expression (13) considers the overall performance of domestic companies (as an industry) as the target to be achieved. Note that we can use the logarithm of this expression, i.e. $\ln U(X.Y) = \sum_{j=1}^{s} \beta_j \ln Y_j - \sum_{i=1}^{m} \alpha_i \ln X_i$ for optimization purposes.

The problem of maximizing the performance indicator (13) is as follows and can be used for computational solving:

$$U(X,Y) \to \max \, , \tag{14}$$

$$X = \sum_{r=1}^{n_d} x_r \ , \tag{15}$$

$$Y = \sum_{r=1}^{n_d} y_r \ , \tag{16}$$

$$(x_r, y_r) \in \Omega \qquad \forall r \in \{1, 2, \ldots, n_d\}, \tag{17}$$

$$x_r \le \overline{x}_r \qquad \forall r \in \{1, 2, \ldots, n_d\}, \tag{18}$$

$$y_r \ge \overline{y}_r \qquad \forall r \in \{1, 2, \ldots, n_d\}, \tag{19}$$

where $x_r = (x_{r1}, \ldots, x_{rm})$, $y_r = (y_{r1}, \ldots, y_{rs})$, $X = (X_1, \ldots, X_m)$, $Y = (Y_1, \ldots, Y_s)$ are variables, and $\overline{x}_r := (\overline{x}_{r1}, \ldots, \overline{x}_{rm})$, $\overline{y}_r := (\overline{y}_{r1}, \ldots, \overline{y}_{rs})$, are known values obtained from collected data.

Conditions (18) and (19) account for the interests of individual companies (banks) while evaluating the overall performance of the industry. The problem defined from (14) to (19) is equivalent to the following formulation, which is more suitable for computational solution:

$$\ln U(X.Y) := \sum_{j=1}^{s} \beta_j \ln Y_j - \sum_{i=1}^{m} \alpha_i \ln X_i \to \max , \tag{20}$$

$$X = \sum_{r=1}^{n_d} x_r \ , \tag{21}$$

$$Y = \sum_{r=1}^{n_d} y_r \ , \tag{22}$$

$$x_r = \sum_{l=1}^{n} \lambda_{rl} \overline{x}_l \ , \ \forall r \in \{1, 2, \ldots, n_d\}, \tag{23}$$

$$y_r = \sum_{l=1}^{n} \lambda_{rl} \overline{y}_l \ , \qquad \forall r \in \{1, 2, \ldots, n_d\}, \tag{24}$$

$$\sum_{l=1}^{n} \lambda_{rl} = 1, \qquad \forall r \in \{1, 2, \ldots, n_d\}, \tag{25}$$

$$\lambda_{rl} \ge 0 \qquad \forall r \in \{1, 2, \ldots, n_d\}, \qquad \forall l \in \{1, 2, \ldots, n\}, \tag{26}$$

$$x_r \le \overline{x}_r \qquad \forall r \in \{1, 2, \ldots, n_d\}, \tag{27}$$

$$y_r \ge \overline{y}_r \qquad \forall r \in \{1, 2, \ldots, n_d\}, \tag{28}$$

where $x_r = (x_{r1}, \ldots, x_{rm})$, $y_r = (y_{r1}, \ldots, y_{rs})$, $X = (X_1, \ldots, X_m)$, $Y = (Y_1, \ldots, Y_s)$, and $\Lambda := [\lambda_{rl}]$ ($r \in \{1, 2, \ldots, n_d\}$, $l \in \{1, 2, \ldots, n\}$) are variables.

The optimal vectors x_r^* and y_r^* can be considered as goals for r^{th} company (bank) (from the view of the whole domestic country's interest), and we can also calculate slack:

$$S_r^- = \overline{x}_r - x_r^* \qquad \forall r \in \{1, 2, \ldots, n_d\}, \tag{29}$$

$$S_r^+ = y_r^* - \overline{y}_r \qquad \forall r \in \{1, 2, \ldots, n_d\}. \tag{30}$$

The optimal vectors X^* and Y^* can be considered as goals for the whole domestic industry, they are used to calculate the slacks for each input and output variable:

$$S^{-} = \overline{X} - X^{*}, \tag{31}$$

$$S^{+} = Y^{*} - \overline{Y}, \tag{32}$$

where $\overline{X} = \sum_{r=1}^{n_d} \overline{x}_r$, $\overline{Y} = \sum_{r=1}^{n_d} \overline{y}_r$.

The model formulated from (20) to (28) is a new formulation and is not a special case of Coopman's models. The LP equations for this model can be solved using Excel. However, due to its larger size and more complex structure, it may be more practical to solve it in AMPL[4], a powerful mathematical programming language. It was used to determine the optimal values of the model's inputs and outputs based on the provided dataset, and the results were then automatically stored for further analysis and interpretation. This approach ensures consistency between data handling, model execution, and output generation.

To estimate the parameters «α_i and β_j» formulated in optimization problem (33)–(36):

$$S := \left(\sum_{j=1}^{s} \beta_j \ln \frac{\hat{Y}_{lj}}{\tilde{Y}_{lj}} - \sum_{i=1}^{m} \alpha_i \ln \frac{\hat{X}_{li}}{\tilde{X}_{li}} \right)^2 \to \min, \tag{33}$$

$$\sum_{i=1}^{m} \alpha_i = 1, \tag{34}$$

$$\alpha_i \geq 0 \,.\, i = \overline{1, m}, \tag{35}$$

$$\beta_j \geq 0, \; j = \overline{1, s}\,. \tag{36}$$

it is necessary to estimate at least $m + s - 1$ pairs of equivalent vectors $\left(\tilde{X}_l, \tilde{Y}_l\right)$ and $\left(\hat{X}_l, \hat{Y}_l\right)$. Ideally, such equivalence would be based on standardized credit ratings (e. g., AA, A+, BBB), allowing for direct matching of banks with similar levels of creditworthiness and financial situation. However, in practice, most Lebanese banks do not have ratings from global credit agencies, making this direct approach infeasible. As a result, the researcher adopted an alternative matching methodology explicitly matching the objective function in (33), this approach directly exploits the structure of the logarithmic input–output differences embedded in the objective. For each bank, we computed the differences defined in (33), namely, the difference between the logarithm of each corresponding ratio of outputs and the logarithm of the ratio of the corresponding inputs, against all other banks in the sample. Importantly, these differences were calculated without imposing any coefficients (i.e., setting all α_i and β□ equal to unity at this stage). For each pair of banks, we then calculated the standard deviation of these differences across all inputs and outputs. Banks were subsequently paired by selecting the four pairs that exhibited the minimum standard deviation of differences, indicating the highest degree of similarity in their input–output structures. This methodology proved successful in yielding strictly positive estimates for all α_i and β□, whereas alternative approaches, such as matching banks based on comparable size, customer deposits, failed to achieve this outcome. Following this methodology, we constructed four pairs of equivalent banks which are «Audi and IBL», «Byblos and BLF», «BLOM and BOB», «Credit Libanais and FNB». The following table summarizes data for the four pairs of equivalent vectors $\left(\tilde{X}_l, \tilde{Y}_l\right)$ and $\left(\hat{X}_l, \hat{Y}_l\right)$.

[4] AMPL (A Mathematical Programming Language) is a high-level modeling language used to formulate and solve optimization problems such as linear, nonlinear, and integer programming.

Table 4

Values for $\left(\tilde{X}_l, \tilde{Y}_l\right)$ and $\left(\hat{X}_l, \hat{Y}_l\right)$

	Inputs			Outputs		Inputs			Outputs	
l	$\tilde{X}_{l1}$	$\tilde{X}_{l2}$	$\tilde{X}_{l3}$	$\tilde{Y}_{l1}$	$\tilde{Y}_{l2}$	$\hat{X}_{l1}$	$\hat{X}_{l2}$	$\hat{X}_{l3}$	$\hat{Y}_{l1}$	$\hat{Y}_{l2}$
1	31956	16447	12922	13267	501	5860	2503	3009	873	113
2	18467	10829	5370	5442	164	11315	4076	2934	4323	80.6
3	26917	21980	5165	7165	512	13346	4735	5966	5759	207
4	9340	4915	3177	3304	83.36	13346	4735	5966	5759	207

Source: compiled by the author based on the researcher estimation criteria for equivalent banks

Excel solver was used to calculate the estimated values of α_i and β_j. These parameters, summarized in Table 5, were computed based on the constructed pairs of equivalent vectors (Table 4):

Table 5

Estimated Values for parameters α_i and β_j

α_1	α_2	α_3	β_1	β_2
0.15	0.7265	0.1235	0.1486	0.1981

Source: compiled by the author.

The optimization process runs as follows; data for the year 2018, for Lebanese and global banks, and the values for α_i and β_j are saved in an Excel file named Data_2018, this file is then accessed by the model2.mod and model2.run applications. The optimization is carried out using AMPL and the optimal values for the three inputs and the two outputs will be saved automatically by the AMPL code in the same excel file Data_2018, This approach ensures a smooth integration between parameter estimation and model implementation. The following tables present the optimal solutions for the inputs and outputs, and the slacks, defined in (29)-(32), for the five inputs and outputs:

Table 6

Optimal values for Inputs/Outputs for Lebanese Banks in 2018 (In Million USD)

Bank	Input_1	Input_2	Input_3	Output_1	Output_2
Audi	31956.02	5039.71	5196.75	25152.96	1127.09
BBAC	6315.02	1453.56	1724.42	4569.42	232.85
BEMO	1307.02	483.01	629.00	728.02	18.90
BLC	3907.79	1114.01	1393.98	2638.90	148.48
BLF	11315.02	2152.86	2401.53	8583.21	407.23
BLOM	26917.02	4334.95	4514.37	21107.85	951.35
BML	1416.02	548.92	722.02	781.97	30.07
BOB	13346.02	2436.92	2676.57	10213.62	478.06
Byblos	18467.02	3153.14	3370.06	14324.54	656.66

Cedrus	877.02	693.00	988.00	204.02	43.20
Credit Bank	3337.98	1775.91	206.02	1958.02	22.58
Credit Libanais	9340.02	1876.64	2134.07	6997.76	338.35
Fenicia	1432.03	571.81	533.02	523.65	14.80
FNB	3872.02	1111.88	1393.59	2608.28	147.65
FB	17023.02	2951.18	3174.51	13165.36	606.30
IBL	5860.02	1389.92	1662.80	4204.16	216.98
MEAB	1866.02	410.01	292.01	1367.02	4.90
MED	13334.02	2435.24	2674.94	10203.98	477.64
SGBL	18699.02	3185.59	3401.48	14510.78	664.75

Source: compiled by the author based on AMPL Solver.

Table 7

Slacks for Inputs-Outputs for Lebanese Banks in 2018 (In Million USD)

Bank	Input_1	Input_2	Input_3	Output_1	Output_2
Audi	0	11407.29	7725.25	11885.96	626.09
BBAC	0	1444.44	901.58	2818.42	181.15
BEMO	0	0	0	0	0
BLC	0	0	426.02	1031.90	107.48
BLF	0	1923.14	532.47	4260.21	326.63
BLOM	0	17645.05	650.63	13942.85	439.35
BML	0	268.08	0	380.97	10.07
BOB	0	2298.08	3289.43	4454.62	271.06
Byblos	0	7675.86	1999.94	8882.54	492.66
Cedrus	0	0	0	0	0
Credit Bank	0	0	0	0	0
Credit Libanais	0	3038.36	1042.93	3693.76	254.99
Fenicia	0	0	0	0	0
FNB	0	601.12	201.41	1661.28	110.90
FB	0	3136.82	3025.49	6425.36	432.30
IBL	0	1113.08	1346.20	3331.16	103.98
MEAB	0	0	0	0	0
MED	0	3043.76	4995.06	6584.98	446.64
SGBL	0	7836.41	2987.52	8700.78	469.75

Source: compiled by the author based on AMPL Solver.

Analysis and Recommendations

The findings from this advanced model provide a precise diagnosis of the Lebanese banking sector regarding their efficiency. According to the EPEOM, only five out of the nineteen banks analyzed were found efficient, with no slack in any of their inputs or outputs. These efficiently operating banks were BEMO, Cedrus, Credit Bank, Fenicia, and MEAB. For the remaining fourteen banks, the analysis of slack variables powerfully confirms the objective of this research: the primary source of inefficiency was the mismanagement of funds associated to excess deposits with the central bank and inefficient investments. The biggest source of inefficiency was found in Input_2, Cash and Deposits at the Central Bank, with 13 out of 19 banks maintaining excess reserves beyond what is required for efficient operation. This misallocation was enormous; for instance, BLOM had a slack of $17.6 billion in this input, while Audi had a slack of $11.4 billion. Closely related was Input_3, Investments, which showed slack in 13 out of 19 banks, directly validating the research question that excessive, non-diversifiable investments in government bonds were a primary driver of poor performance. The consequences of these input inefficiencies are clearly reflected in the outputs, where 13 out of 19 banks showed slack in Loans and Advances to Customers, and 14 out of 19 showed slack in Net Profit, proving that banks failed to translate their vast resource base into productive lending and maximum profitability.

This model introduces a novel approach to evaluating the efficiency of banks by combining elements from optimization theory, production economics, and econometrics. Unlike traditional efficiency models, it incorporates industry-wide constraints, providing a dual-level analysis that captures firm-level and sector-level dynamics. A key innovation is its use of convex production sets and elasticities of substitution, allowing the model to realistically reflect how banks can reallocate resources in response to changing conditions, rather than assuming rigid production relationships. It also confirms that the “competitive gap” observed against global peers was not merely a relative phenomenon, but a reflection of deep, absolute inefficiencies inherent in the strategies of the largest Lebanese banks, inefficiencies that only a model as sophisticated as the EPEOM can reveal it.

Another important novelty lies in the model’s use of logarithmic transformations within its optimization framework. This multiplicative structure departs from the typical additive formulations seen in standard DEA models, enabling a more flexible and theoretically sound representation of efficiency, especially when dealing with varying scales of operation across banks. The transformation simplifies the mathematical problem while preserving important economic relationships, making the performance indicators more robust to differences in size and structure among financial institutions. The model further distinguishes itself by integrating parameter estimation directly into the optimization process. The model also blends optimization with econometric techniques to estimate key parameters (such as productivity coefficients) within the model itself, which represents a significant step beyond conventional efficiency analysis, where parameter values are typically assumed or externally estimated. Finally, from a computational standpoint, the model’s design for large-scale implementation using tools like AMPL, rather than relying solely on spreadsheet software like Excel, marks another key advancement. This makes the model scalable and applicable to real-world financial systems with complex datasets.

The financial collapse that began in Lebanon in late 2019 will not diminish the importance of the findings of this study. Despite the selective default of Lebanese banks in paying back the customers’ deposits in foreign currencies, none of those banks declared bankruptcy, therefore, the results of this study remain a valid benchmark and reference for banks that could be used to recognize their deficiencies in using the available inputs to produce the desired outputs efficiently, and to avoid committing the same mistakes. This research also contributes to the academic literature by introducing and applying a novel modeling approach in the analysis of banking efficiency. It provides a valuable reference for future studies seeking to explore similar financial contexts or extend the proposed methodology. Moreover, the findings offer practical insights not only for Lebanese banks but also for banks in other countries, helping them avoid the structural and strategic mistakes observed in the Lebanese banking sector.

For Lebanese banks to overcome their current liquidity problems, and to adjust their previous investment strategies, they must adhere to more strict borrowing and lending policies, however, banks can't overcome the crisis without a stable monetary policy, a financial restructuring model, and an economic recovery plan. The Lebanese government must coordinate with the Central Bank of Lebanon to take the initiative to establish and implement these reforms in cooperation with banks and other vital sectors in Lebanon. These measures help to relieve the pressure on banks, create an appropriate investment environment, and restore the intermediary role for banks. At the macroeconomic level, adopting a credible economic recovery plan, supported the International Monetary Fund, is essential to rebuild confidence, attract foreign investments, and secure financial assistance. This should be accompanied by fiscal reforms aimed at reducing the budget deficit, restructuring public debt, particularly Eurobonds, and improving governance and efficiency in the public sector. On the monetary side, transitioning toward a more flexible exchange rate regime, limiting dollarization of deposits to fresh dollar inflows, and aligning interest rate policies with economic fundamentals are crucial to restoring monetary stability and competitiveness. At the banking level, restructuring measures must include rigorous solvency stress tests, followed by appropriate actions such as recapitalization, mergers, or liquidation of non-viable institutions. Banks should also adopt prudent risk management practices, diversify their investment portfolios away from excessive exposure to sovereign debt, and comply with international regulatory frameworks.

Conclusion

The outbreak of the Lebanese financial crisis in 2019 underscored the urgent need for a comprehensive and data-driven restructuring of the banking sector. A credible reform strategy must be grounded in a clear assessment of bank efficiency and a critical evaluation of past investment decisions. In this context, this study provides a pioneering contribution by examining the efficiency of Lebanese banks through the lens of their exposure to public debt, an area that has remained largely unexplored in the academic literature. The findings offer practical insights that can support policymakers and stakeholders in assessing bank solvency and their capacity to withstand and recover from systemic shocks.

The originality of this research lies not only in its focus but also in its methodological design. By selecting input and output variables that explicitly capture the implications of sovereign exposure, the study departs from conventional approaches and directly addresses the structural weaknesses underlying bank performance. This contributes to a deeper understanding of the role of risk concentration and highlights the importance of sound risk management practices in safeguarding financial stability. From a methodological perspective, the adoption of the Elasticity-Based Production Efficiency Optimization Model (EPEOM) in its logarithmic form represents a significant advancement in efficiency analysis. The multiplicative structure of the model allows for a more flexible and theoretically consistent representation of production relationships, especially in the presence of heterogeneous bank sizes and operational scales. By integrating optimization techniques with econometric estimation within a unified framework, the model enhances both analytical rigor and empirical reliability. Furthermore, its ability to incorporate industry-level constraints alongside firm-level behavior provides a more comprehensive view of efficiency dynamics within the banking sector. The scalability of the model, supported by implementation through advanced computational tools, further reinforces its applicability to complex financial systems.

The empirical results point to the existence of deep-rooted structural inefficiencies within the Lebanese banking sector prior to the crisis. These inefficiencies are not merely relative when compared to international peers, but reflect fundamental distortions in resource allocation, primarily driven by excessive exposure to government bonds. Such concentration limited banks' ability to perform their core intermediation function effectively, constraining lending activity and weakening profitability. As a result, the sector's vulnerability was amplified, leaving it ill-prepared to absorb the shocks that materialized in the outbreak of the 2019 financial crisis in Lebanon.

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ОЦЕНКА ЭФФЕКТИВНОСТИ ЛИВАНСКИХ БАНКОВ С ИСПОЛЬЗОВАНИЕМ МОДЕЛИ ОПТИМИЗАЦИИ ЭФФЕКТИВНОСТИ ПРОИЗВОДСТВА НА ОСНОВЕ ЭЛАСТИЧНОСТИ (EPEOM)

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Аннотация. В данном исследовании оценивается эффективность ливанских коммерческих банков до финансового кризиса 2019 года, с акцентом на структурные недостатки, которые способствовали краху сектора. Используя данные за 2018 год по 19 банкам и сравнивая их с показателями 7 международных аналогов, исследование представляет новую методологическую основу, выходящую за рамки традиционных методов измерения эффективности. Оно также восполняет пробел в литературе, предоставляя количественную оценку эффективности ливанских банков до кризиса. В исследовании разработана модель оптимизации эффективности производства на основе эластичности (EPEOM), которая интегрирует теорию производства и эластичность замещения для оценки эффективности банков. Результаты показывают значительную неэффективность, обусловленную чрезмерной зависимостью от государственного финансирования и неэффективным распределением ресурсов. Эти выводы предоставляют важную информацию для политиков, стремящихся разработать эффективные стратегии реструктуризации банковского сектора.

Ключевые слова: ливанские банки, измерение эффективности, финансовый кризис, государственный долг, распределение ресурсов, оптимизационная модель, теория производства, эластичность замещения.

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